Development and Application of Acoustic Metamaterials with Locally Resonant Microstructures

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Wave Propagation in Elastic Solids With Negative Mass Density or Modulus

What would happen if mass or modulus becomes negative?

- Dispersion equation: $$q = \omega\sqrt{-\frac{\rho}{E}} = i\beta\omega$$

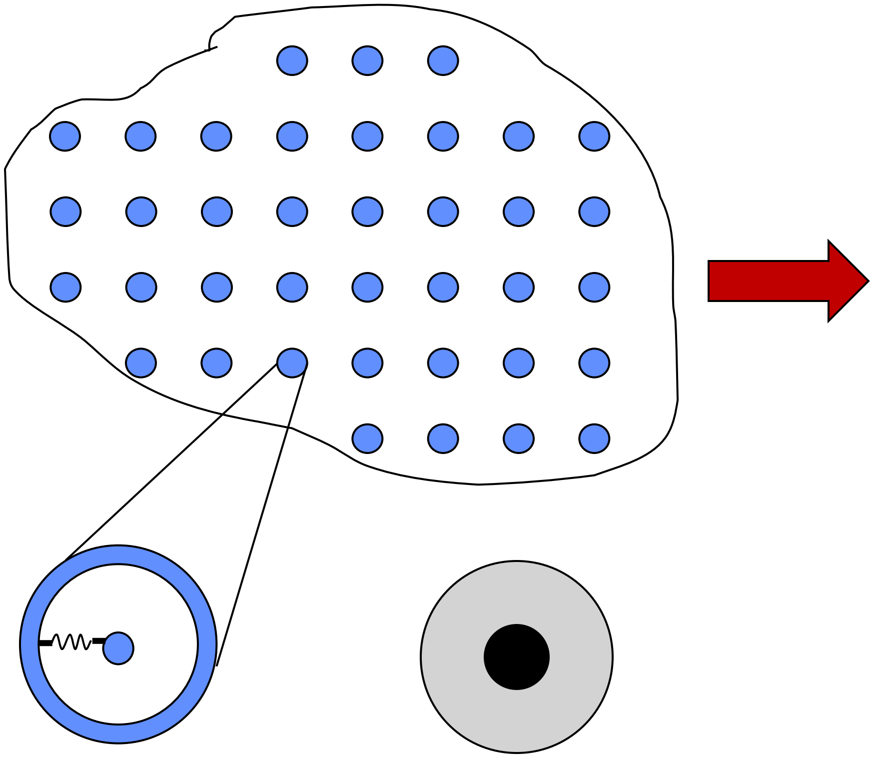
- Wave attenuates: $$u = Ae^{i(qx-\omega t)} = Ae^{-\beta\omega x}e^{i\omega t}$$

β **Is attenuation factor**

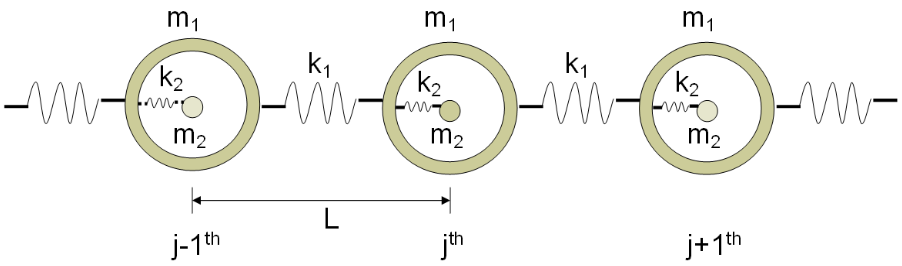
Wave cannot propagate without attenuation in elastic solids with negative mass density or modulus

Metamaterials with Local Resonators

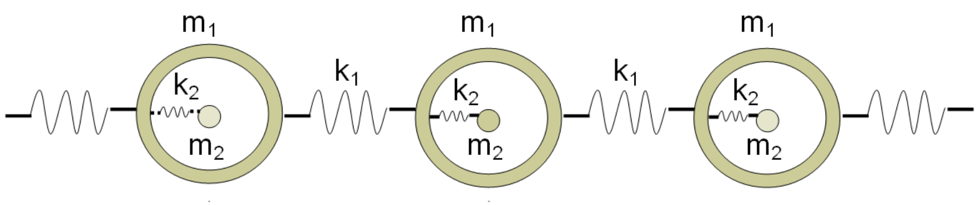
Composite with Resonators



1D Lattice Model



Metamaterials with Negative Effective Mass



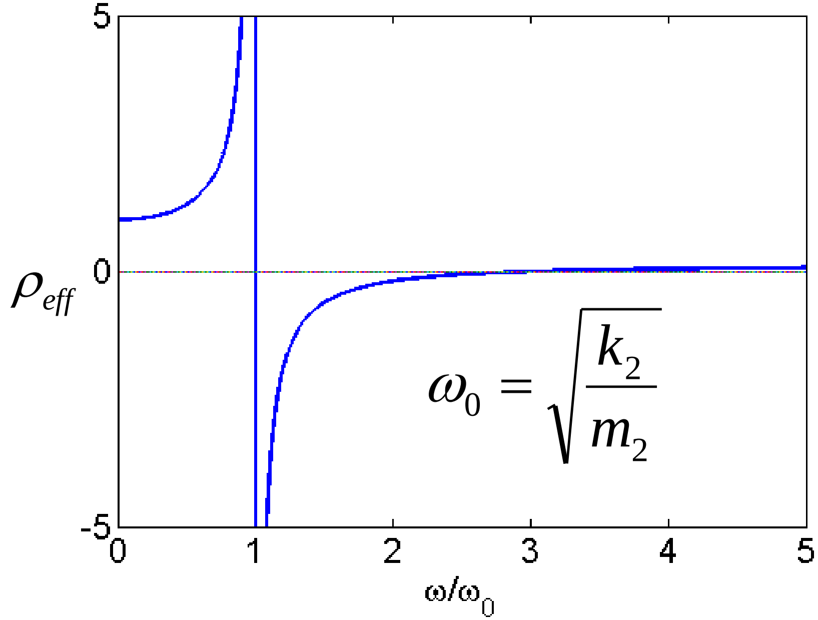
ρ_{eff} , E_{eff}

$$u = Be^{i(qx-\omega t)}$$

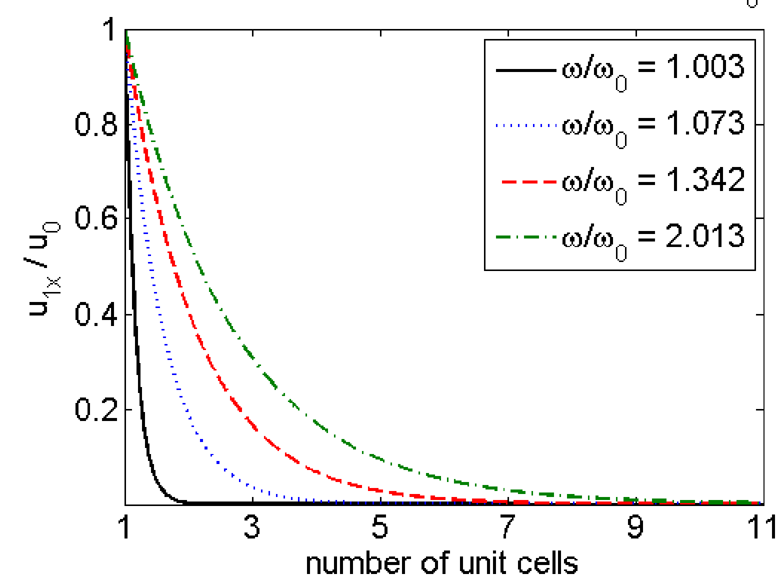
Negative effective mass

Wave attenuation

Effective mass for mass-in-mass lattice

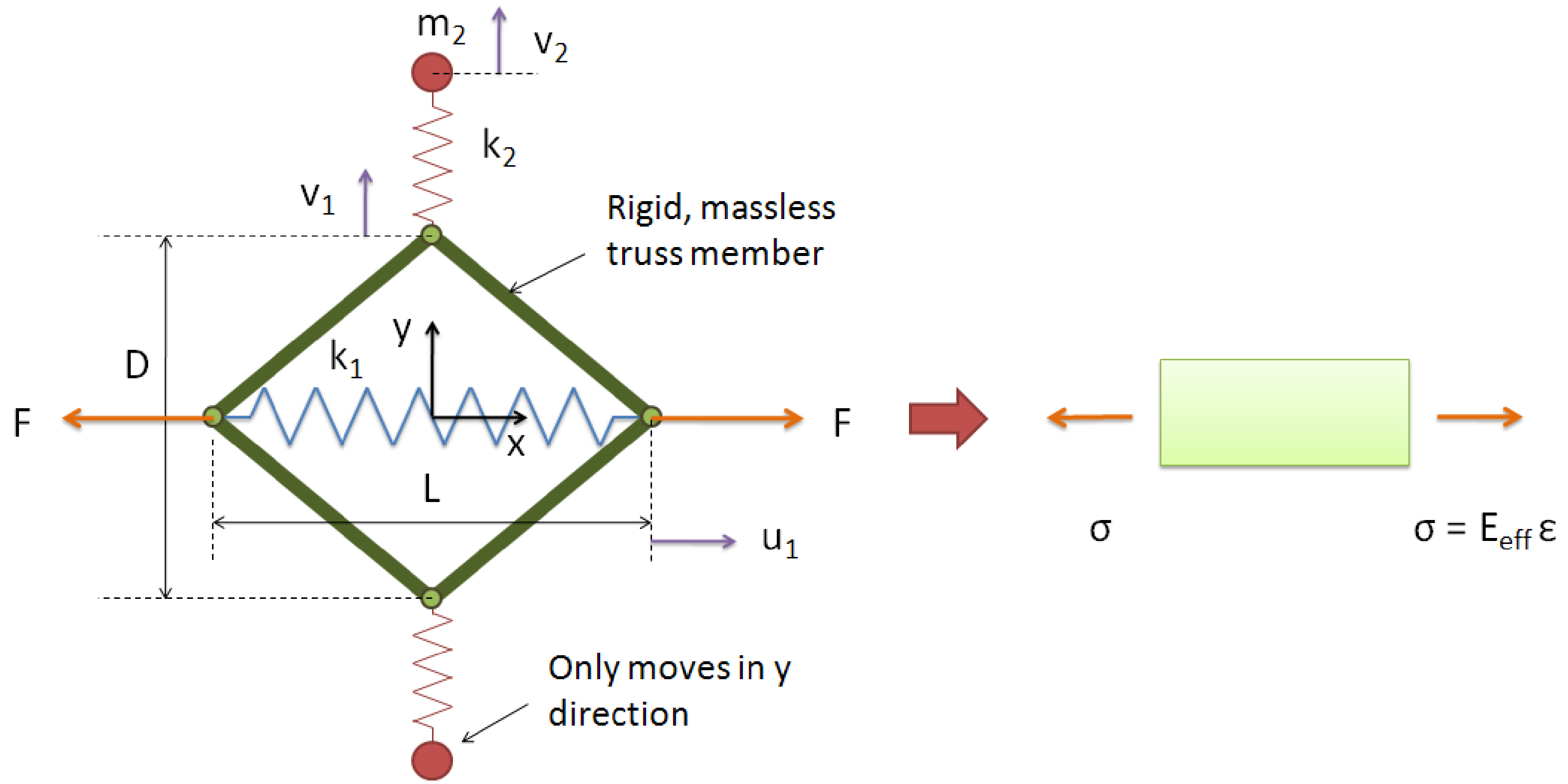


Displacement Envelope, (Local resonance ω_0)

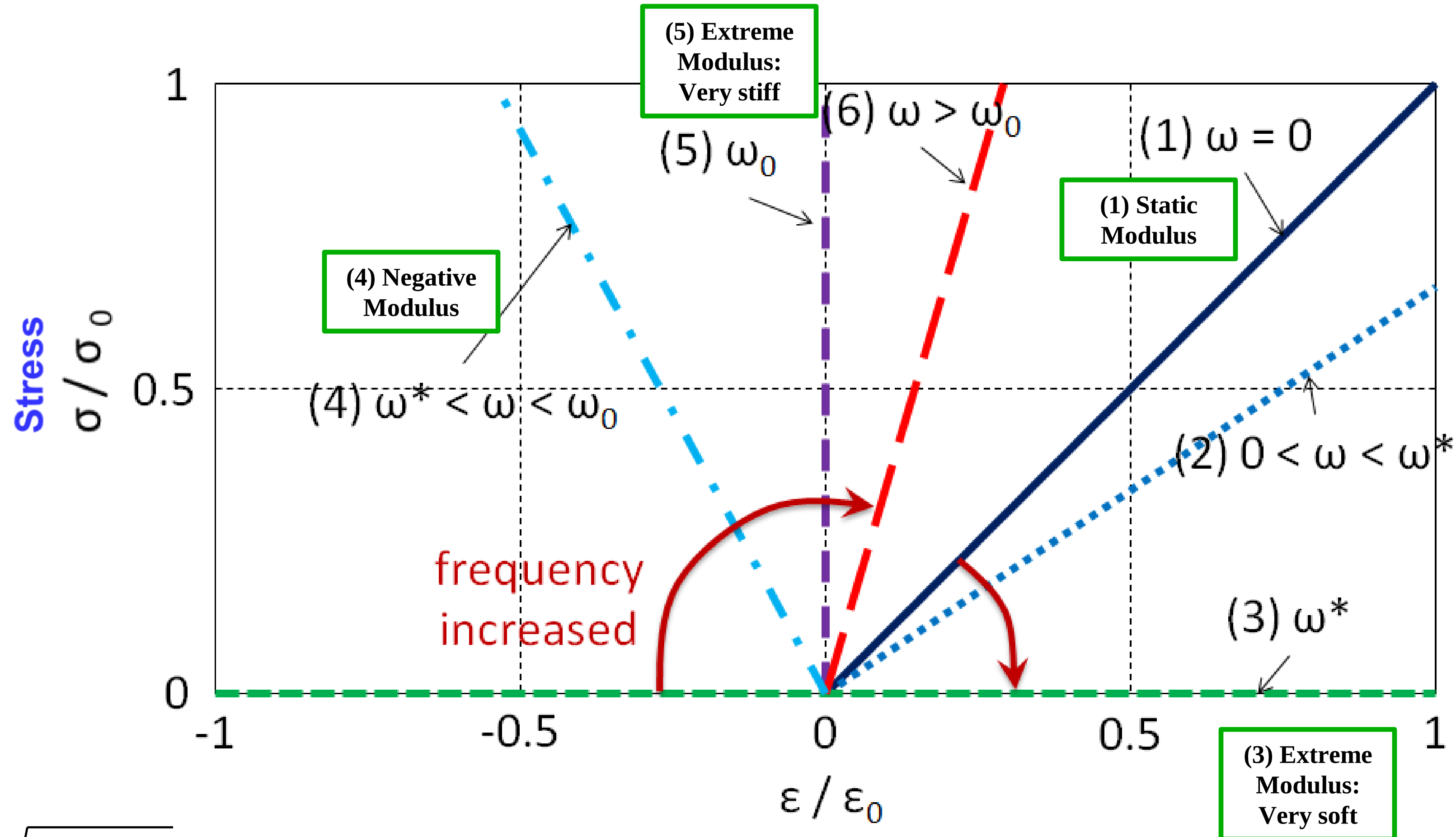


Acoustic Metamaterial with Negative Effective Young's Modulus

A Mechanical Unit Model and Its Representative Elastic Solid



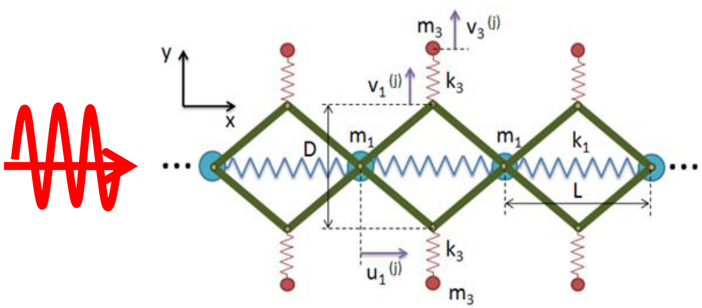
Frequency-dependent Modulus (stress-strain curves)

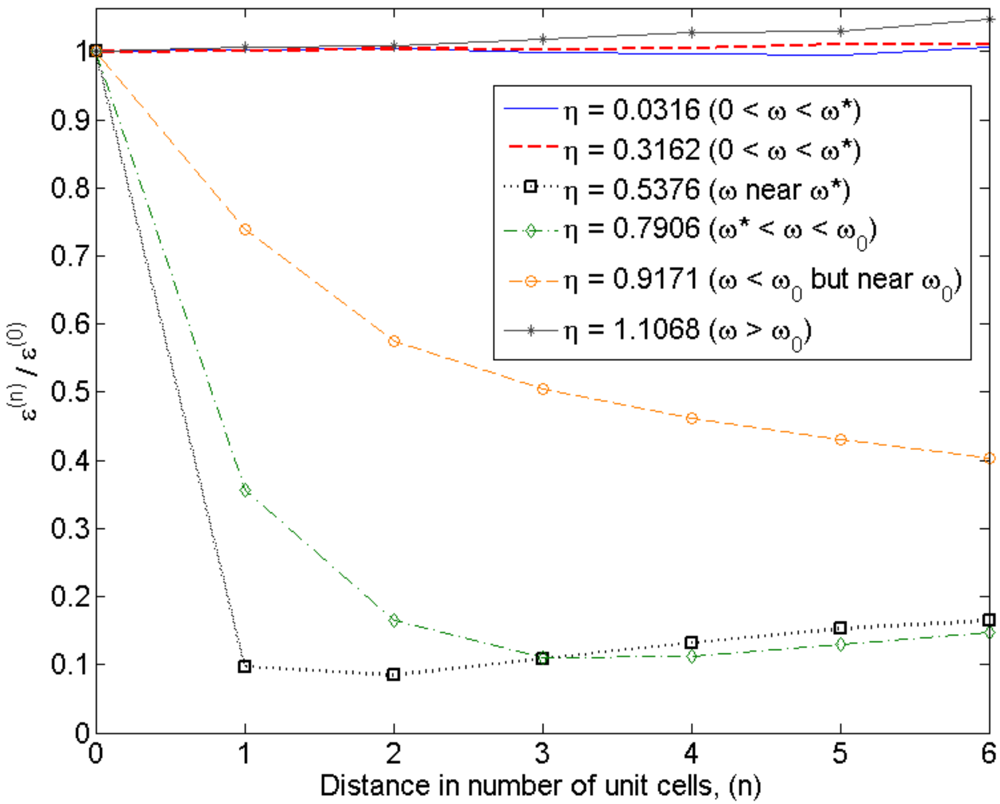


$\omega_0 = \sqrt{k_2 / m_2}$ **Is local resonance frequency**   **Strain**

Wave Attenuation in Metamaterial with Negative Effective Modulus

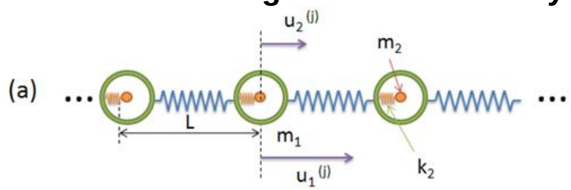
- Wave amplitude decays when its frequency falls inside the band gap, especially if frequency is near the frequency



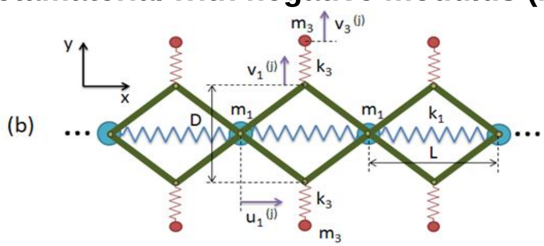


Metamaterial with Double Negativity (DN)

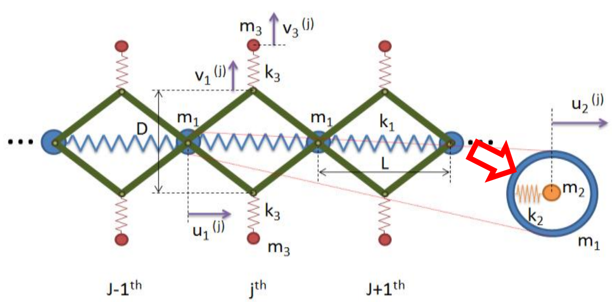
Metamaterial with negative mass density (NMD)



Metamaterial with negative modulus (NEM)



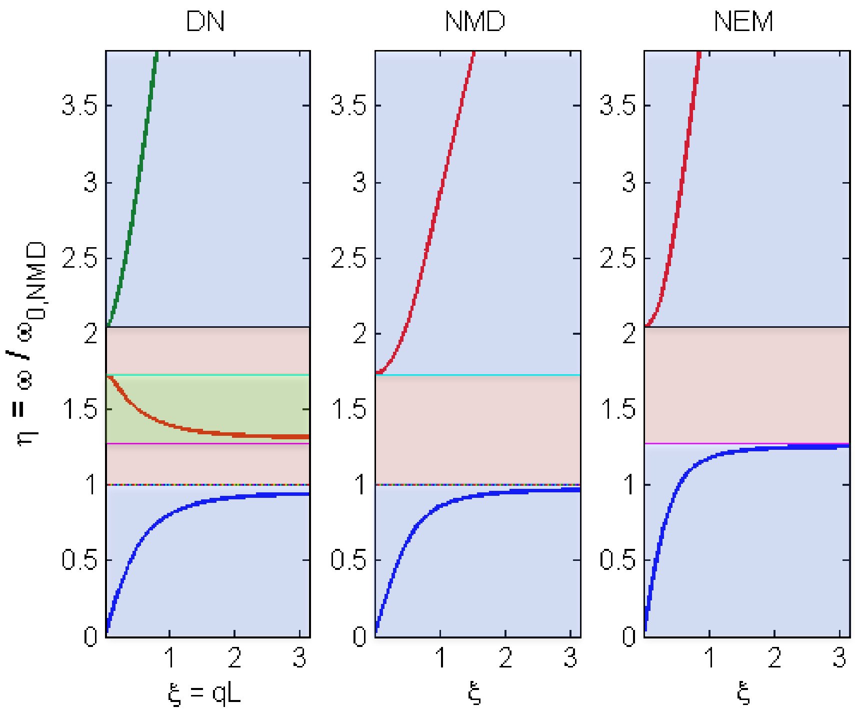
Metamaterial with Double Negativity (DN)



Double Negativity (green area)

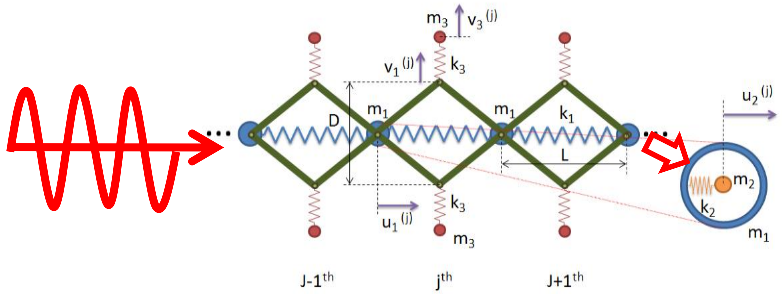
Negative effective mass (Band Gap, red area)

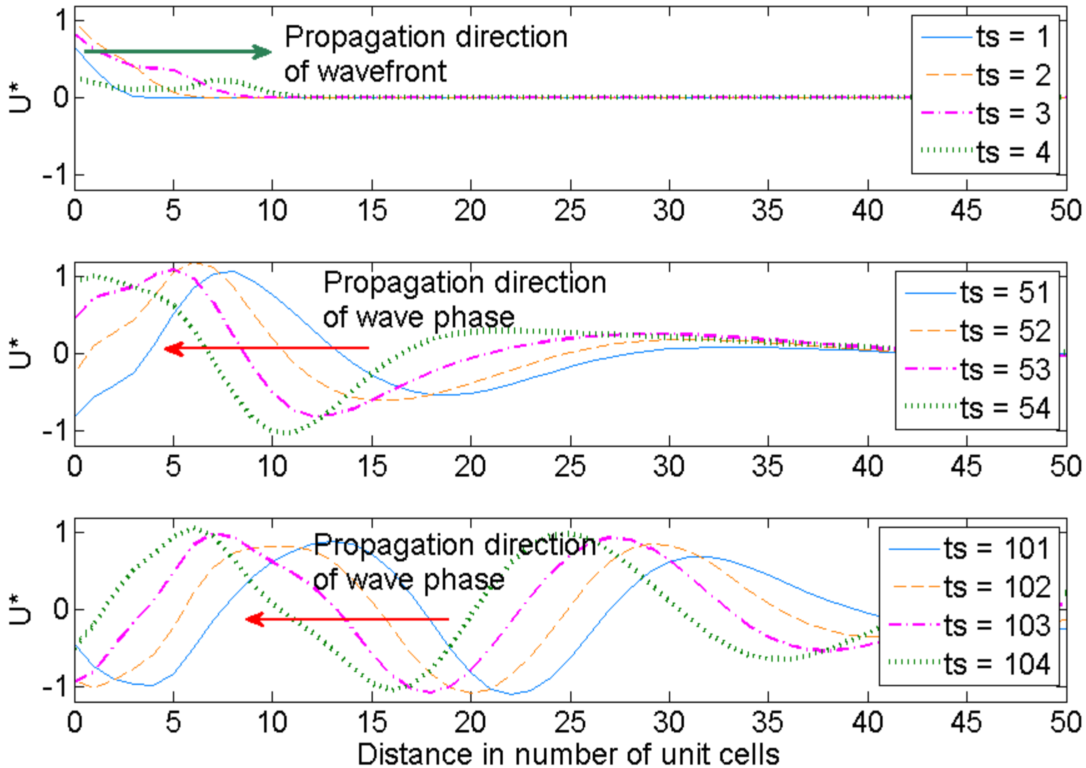
Negative effective modulus (Band Gap, red area)



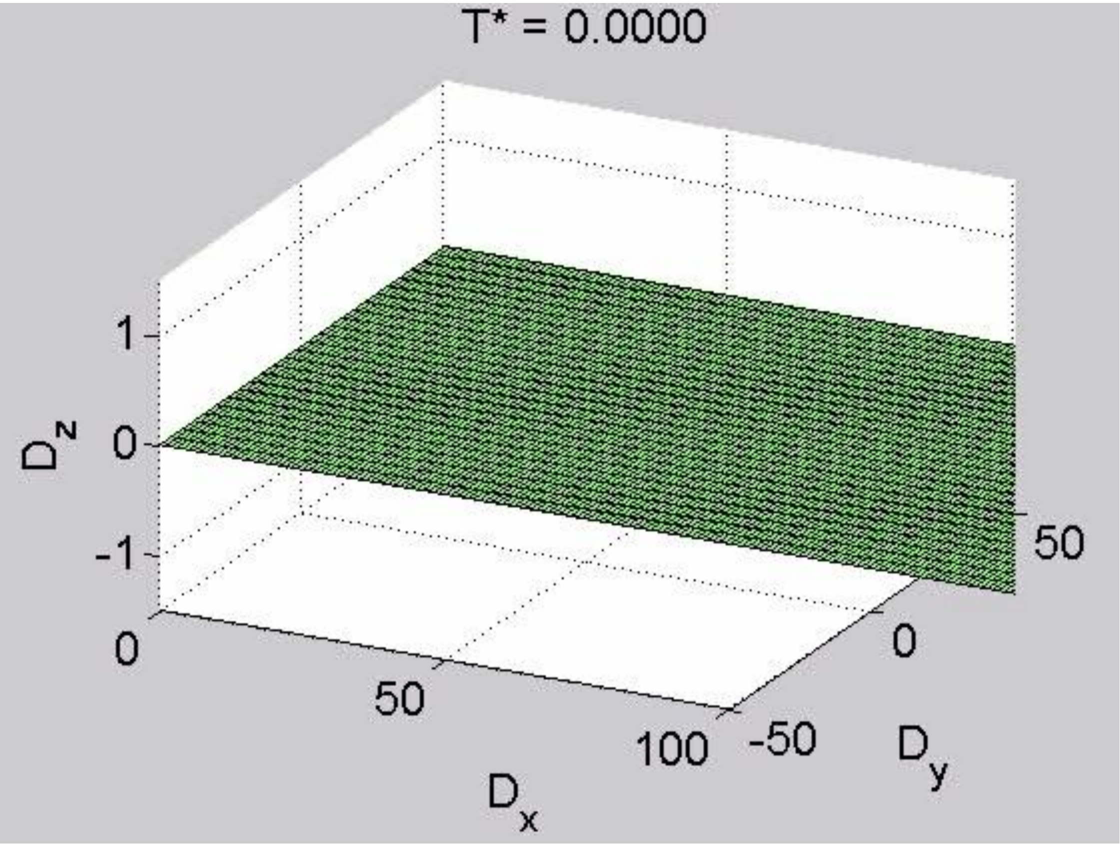
Wave Propagation in Metamaterial with Double Negativity

$$\omega / \omega_0^{NMD} = 1.56$$

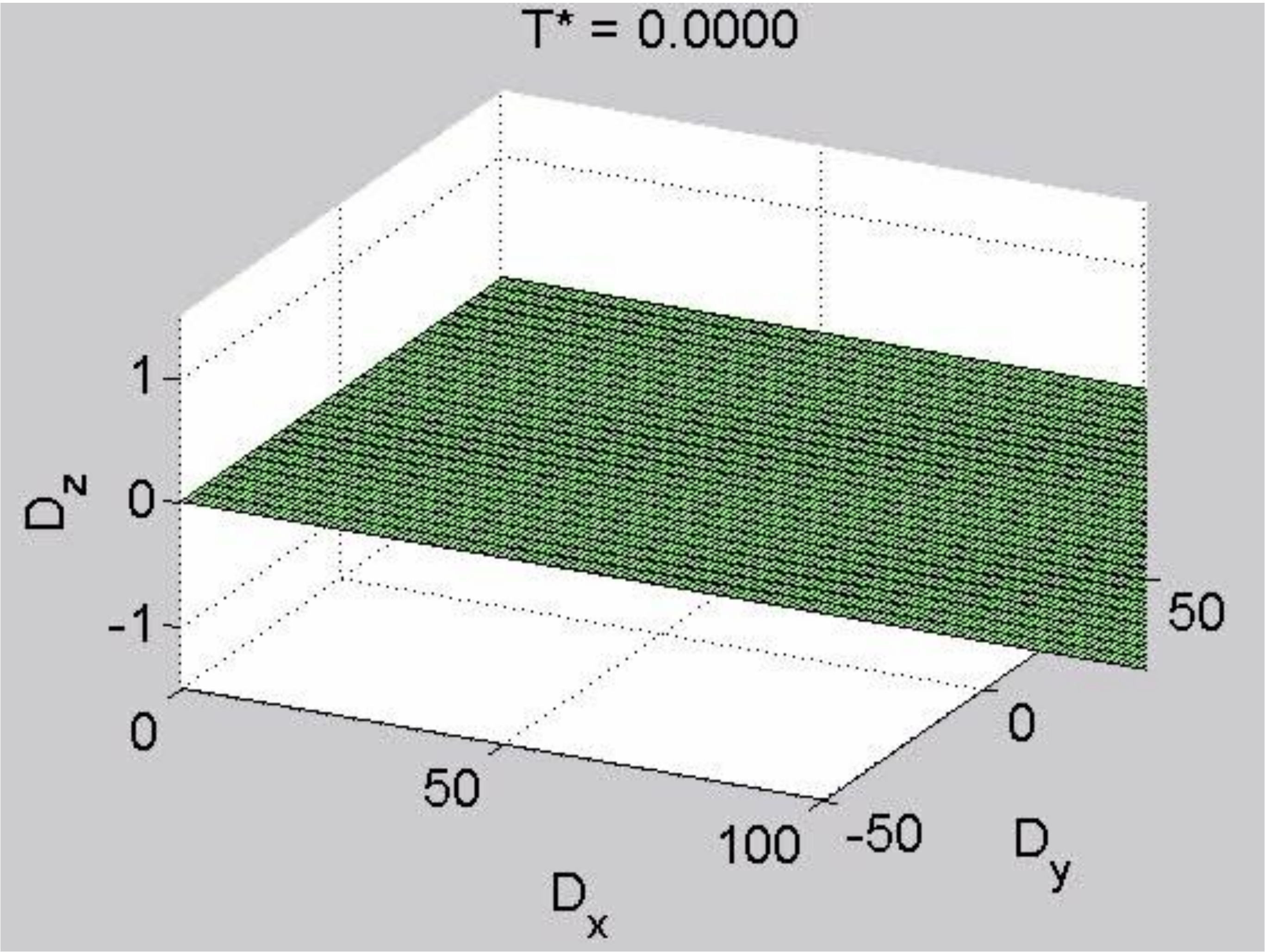




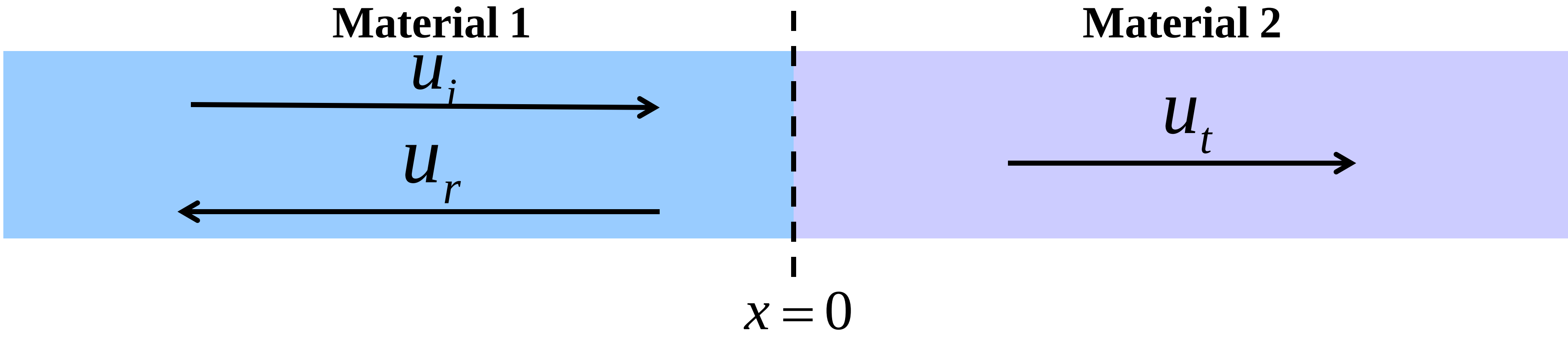
Double Positive Metamaterial



Double Negative Metamaterial



Derivation for Reflection and Transmission Coefficients



Assume

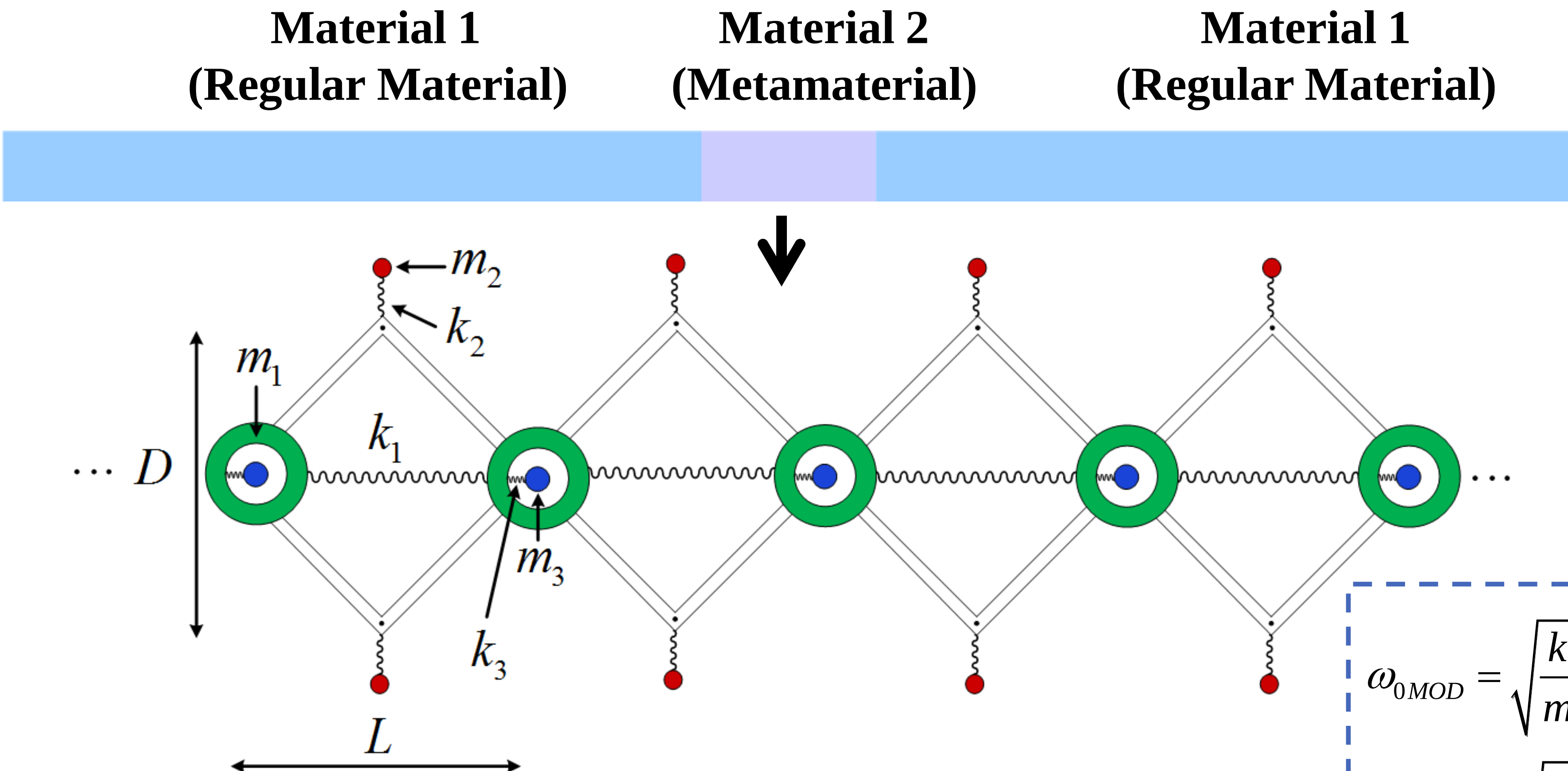
$$u_i = \hat{u}_i e^{i(\omega t - q_1 x)}$$
$$u_r = \hat{u}_r e^{i(\omega t + q_1 x)}$$
$$u_t = \hat{u}_t e^{i(\omega t - q_2 x)}$$

$$R \equiv \frac{\hat{u}_r}{\hat{u}_i} = \frac{E_1 q_1 - E_2 q_2}{E_1 q_1 + E_2 q_2}$$

$$T \equiv \frac{\hat{u}_t}{\hat{u}_i} = \frac{2E_1 q_1}{E_1 q_1 + E_2 q_2}$$

$$\Rightarrow \begin{cases} If\ \ E_1 = E_2,\ \rho_1 = \rho_2,\ then\ R = 0, T = 1 \\ \boxed{If\ \ E_2 = -E_1,\ \rho_2 = -\rho_1,\ then\ R = 0, T = 1} \end{cases}$$

Material 2



$$E_{eff} = \frac{L}{A}\left[k_1 + \left(\frac{1}{2}\right)\left(\frac{k_2\omega^2}{\omega^2 - \omega_{0MOD}{}^2}\right)\left(\frac{L}{D}\right)^2\right]$$

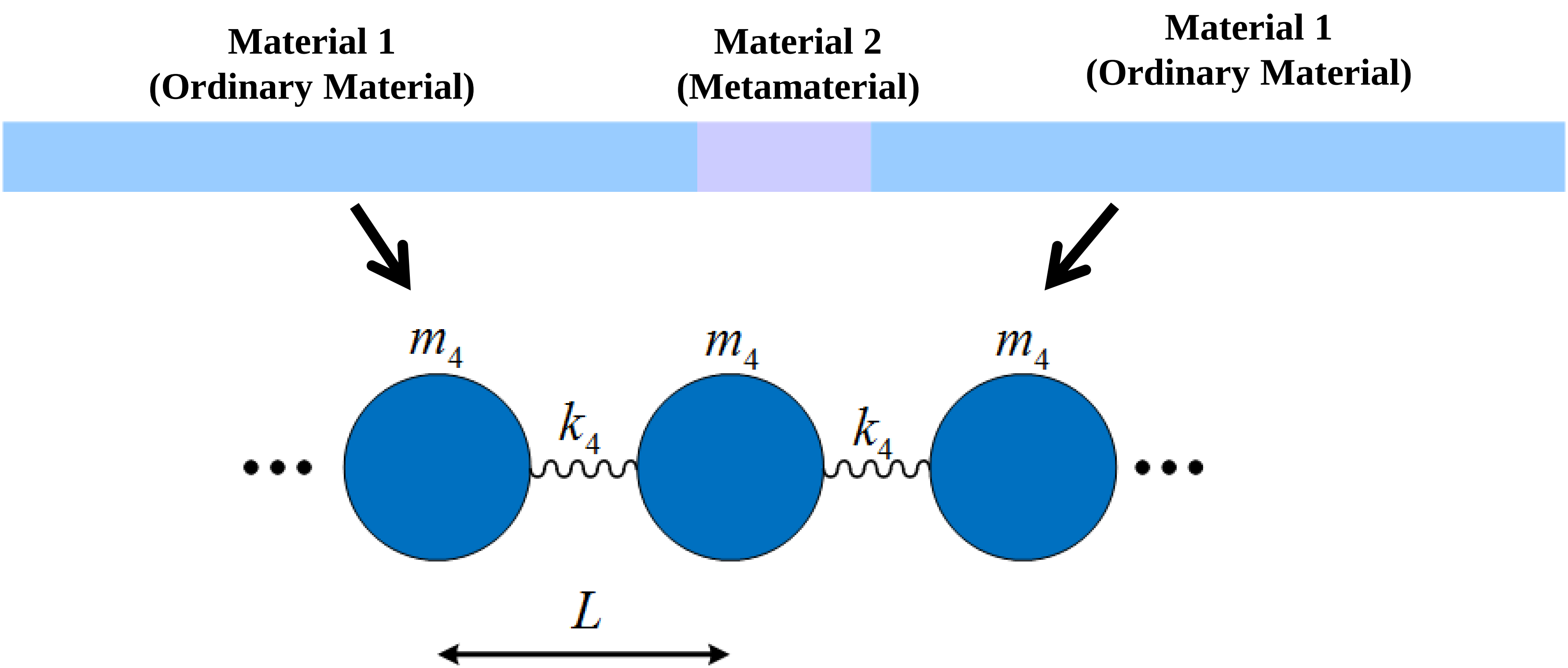
$$\rho_{eff} = \frac{1}{AL}\left[m_1 + m_3\left(\frac{\omega_{0MASS}^2}{\omega_{0MASS}^2 - \omega^2}\right)\right]$$

$$\omega_{0MOD} = \sqrt{\frac{k_2}{m_2}}$$

$$\omega_{0MASS} = \sqrt{\frac{k_3}{m_3}}$$

$$A = 1$$

Material 1

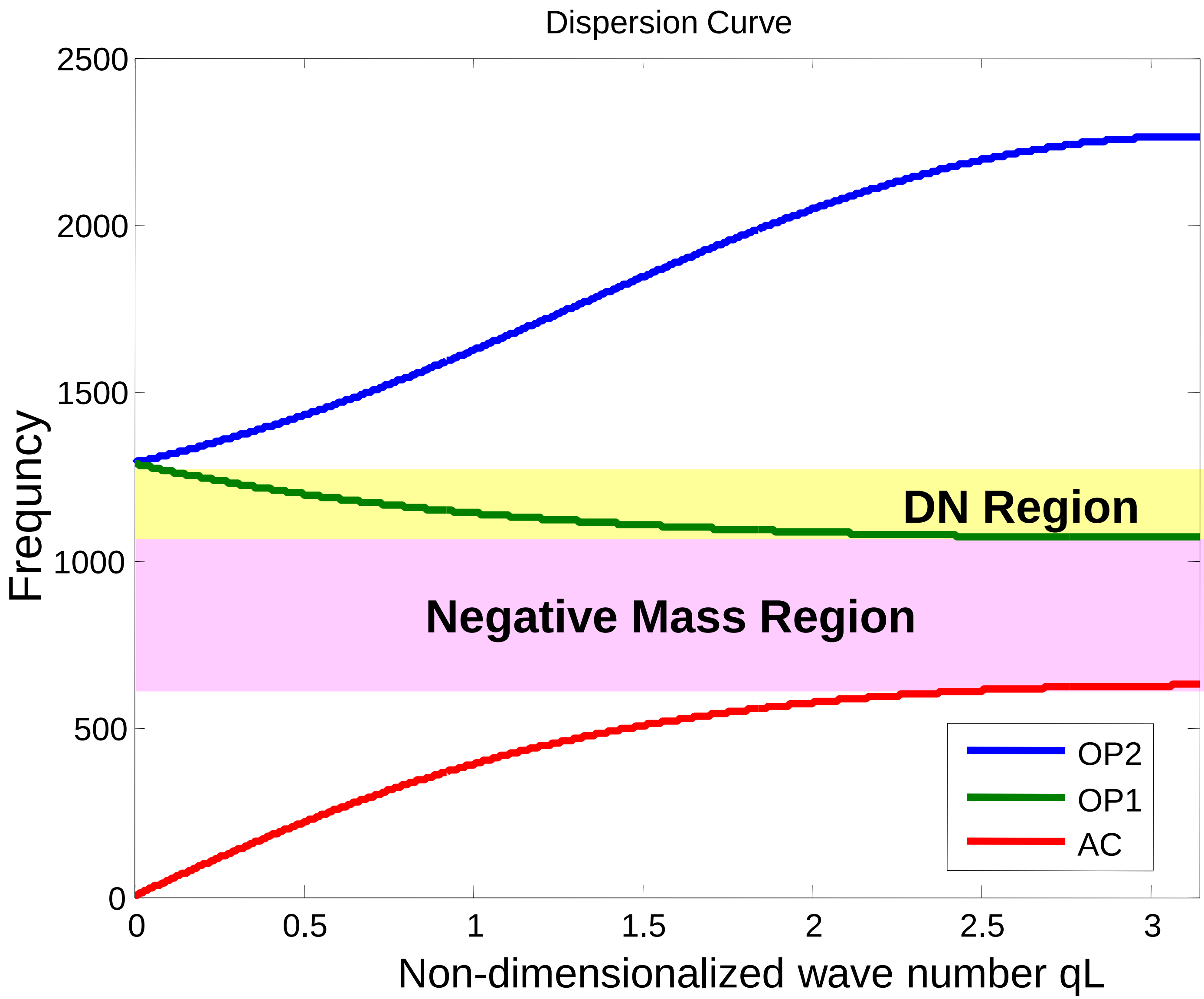


$$E_{eff} = \frac{k_4 L}{A}$$

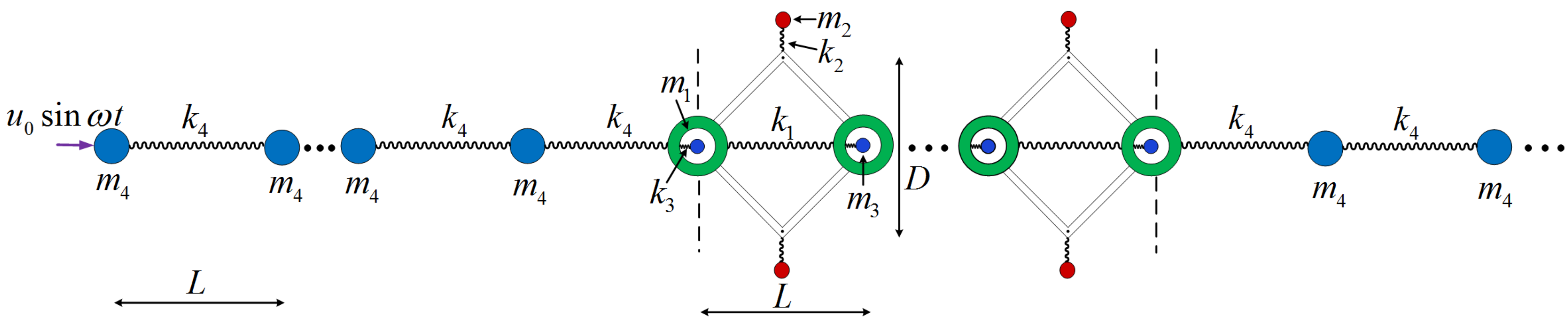
$$\rho_{eff} = \frac{m_4}{AL}$$

where $A=1$

Dispersion Curve for Metamaterial



Material Design



$$Case\,1: \omega = 1200\ (rad\ /\ s) \longrightarrow \text{Frequency for double negativity}$$

$$Case\,2: \omega = 650\ (rad\ /\ s) \longrightarrow \text{Frequency for negative mass}$$

$$m_1 = 2.4 \times 10^{-4}\ (kg) \qquad k_1 = 100.0\ (N\ /\ mm)$$

$$m_2 = 1.2 \times 10^{-4}\ (kg) \qquad k_2 = 200.0\ (N\ /\ mm)$$

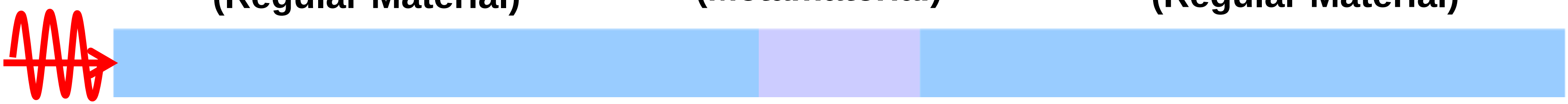
$$m_3 = 2.4 \times 10^{-4}\ (kg) \qquad k_3 = 200.0\ (N\ /\ mm)$$

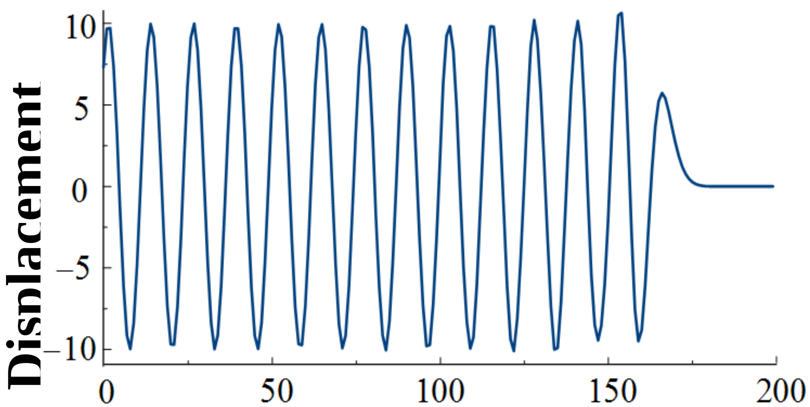
$$m_4 = 9.0 \times 10^{-5}\ (kg) \qquad k_4 = 535.3\ (N\ /\ mm)$$

Case 1: Simulation Result in DN Region

$$\boxed{\omega = 1200\ rad / s}$$

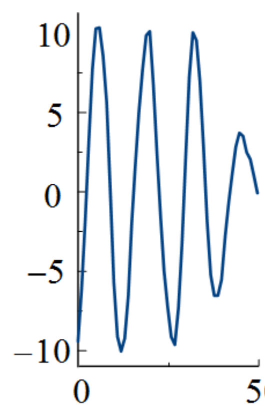
Material 1 (Regular Material) | **Material 2 (Metamaterial)** | **Material 1 (Regular Material)**





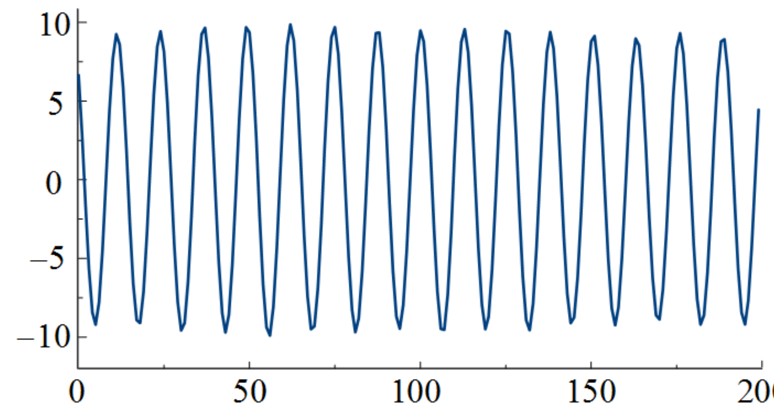
Distance in number of unit cells

$t = 0.07\ s$



Distance in number of unit cells

$t = 0.390\ s$



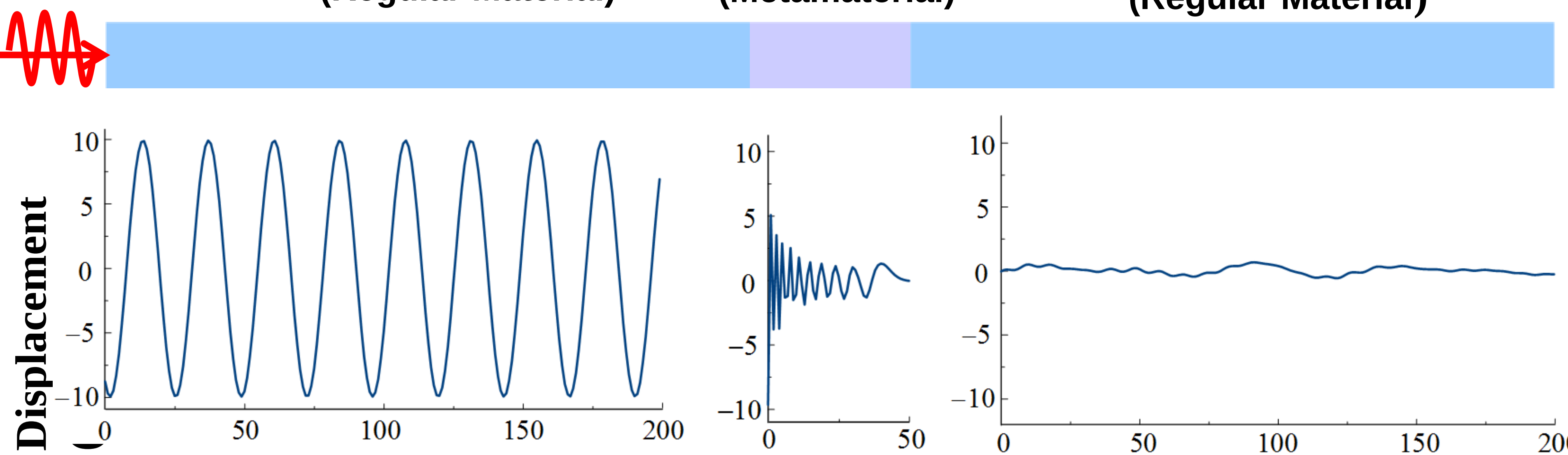
Distance in number of unit cells

$t = 0.775\ s$

Case 2: Simulation Result in Negative Mass Region

$$\boxed{\omega = 650\ rad / s}$$

Material 1 (Regular Material) | **Material 2 (Metamaterial)** | **Material 1 (Regular Material)**



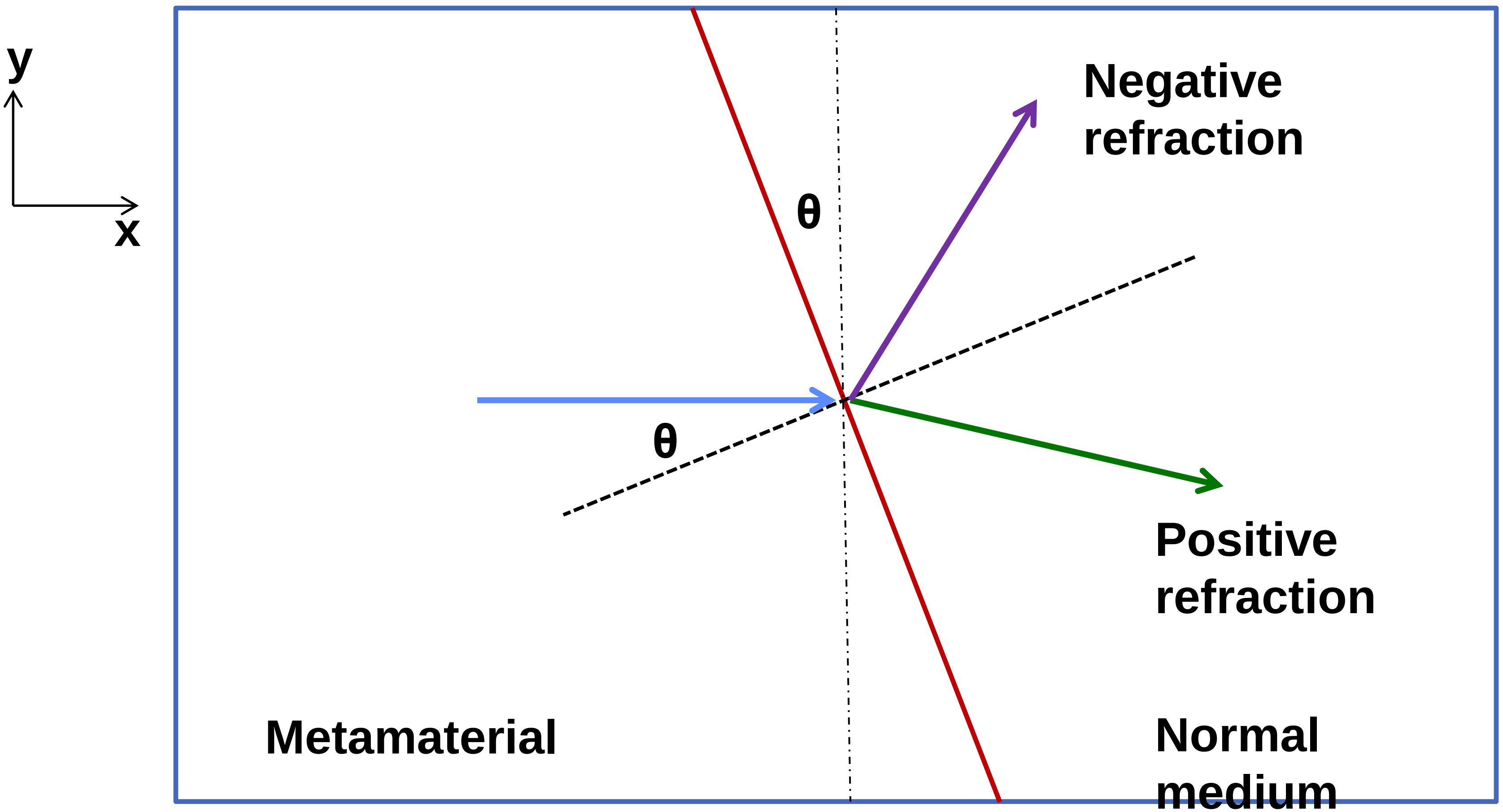
Distance in number of unit cells

Distance in number of unit cells

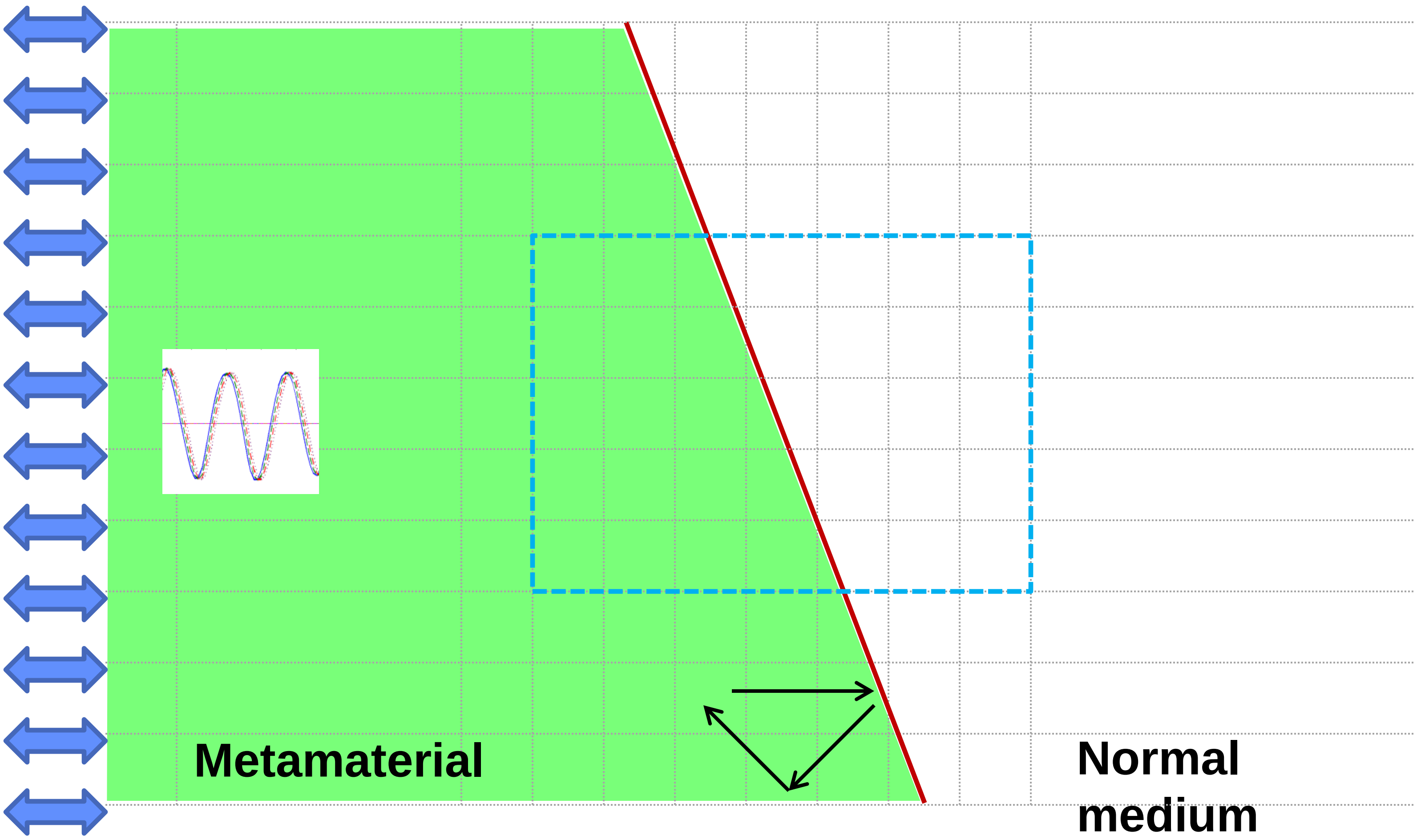
$t = 0.182\ s$ $\quad$ $t = 0.301\ s$ $\quad$ $t = 1.857\ s$

Refraction of Metamaterials

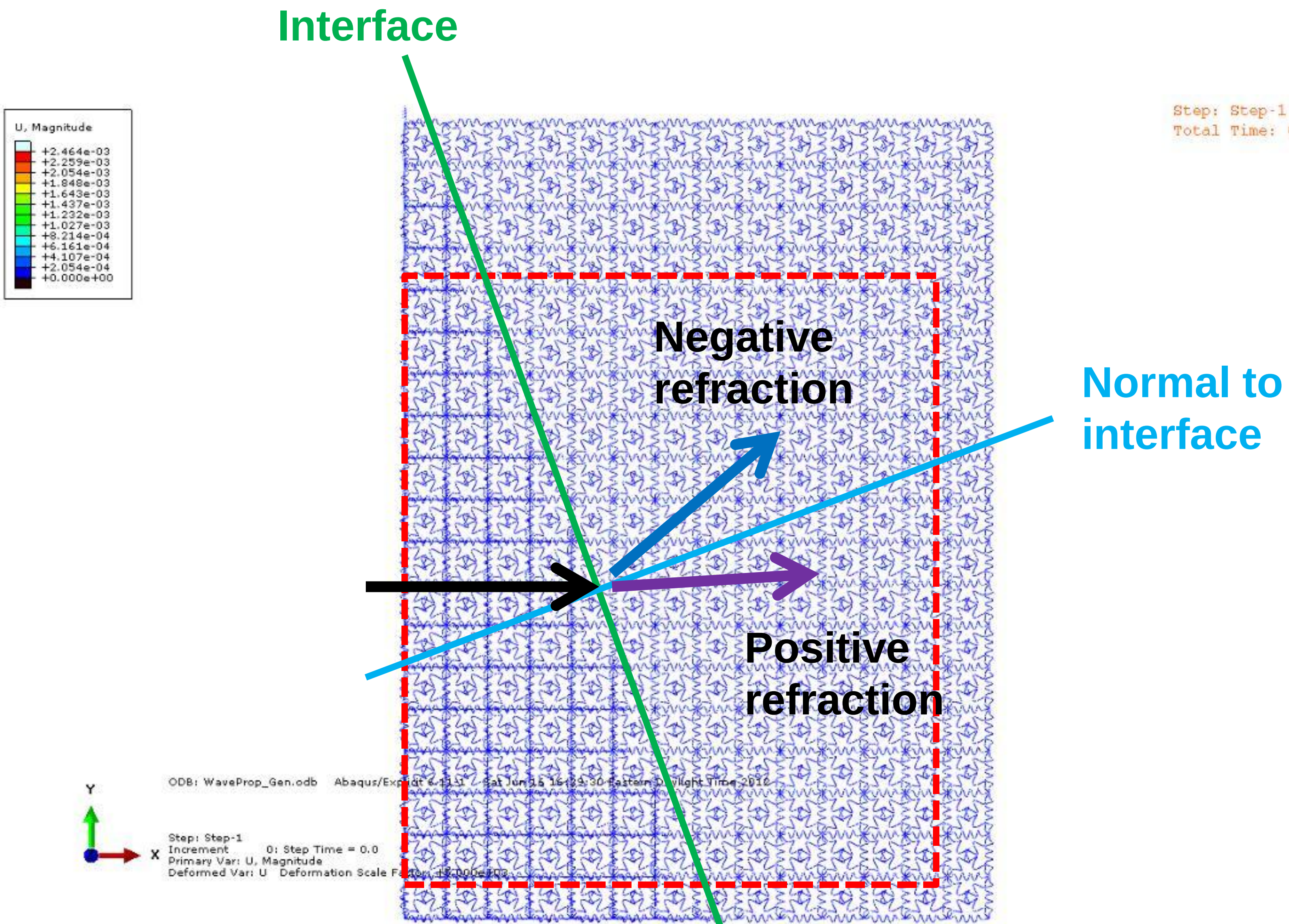
- 2D Double-Negativity Metamaterial

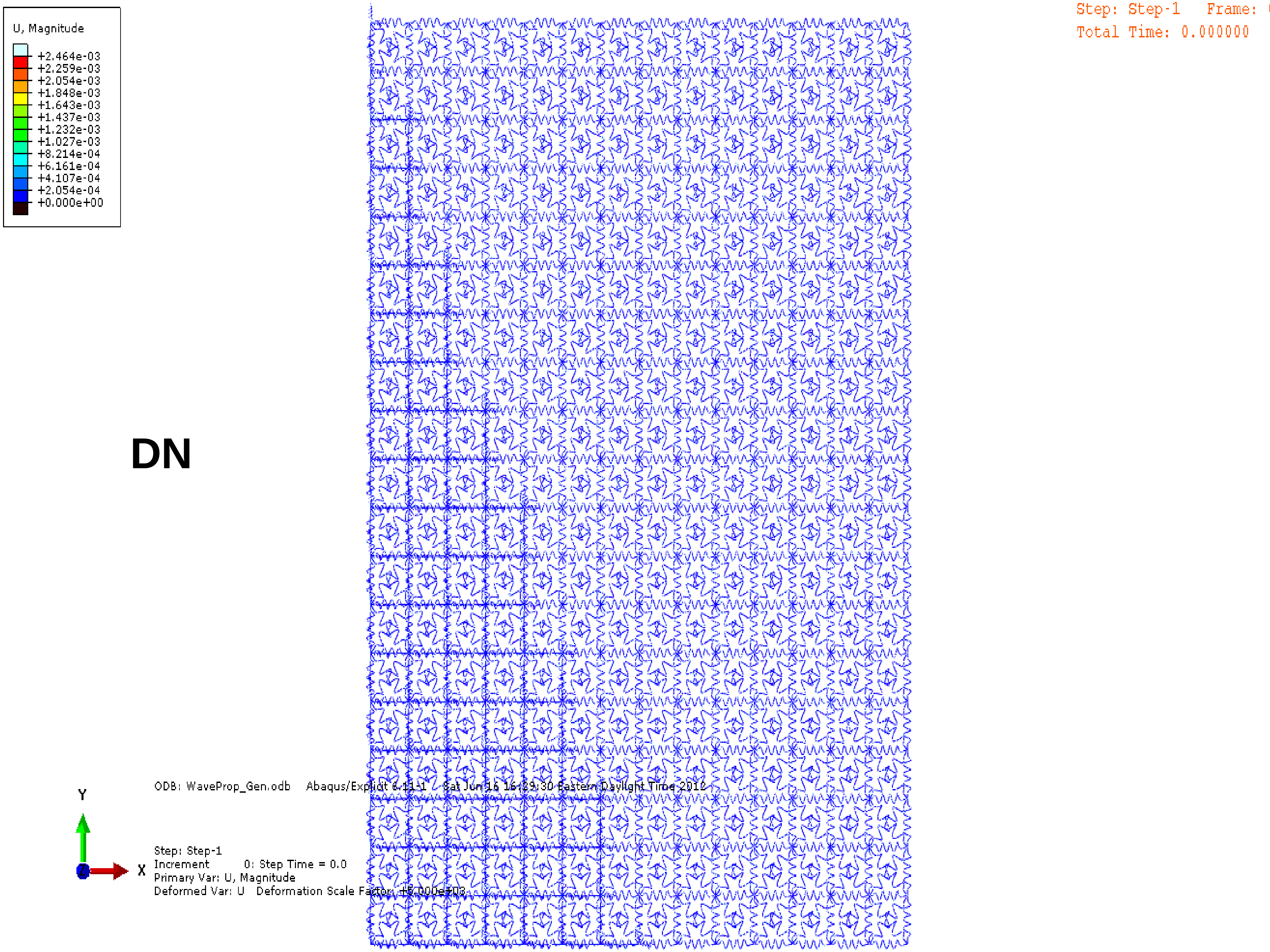


Boundary Condition: Plane wave



Simulation Window (15x20 units)




U, Magnitude
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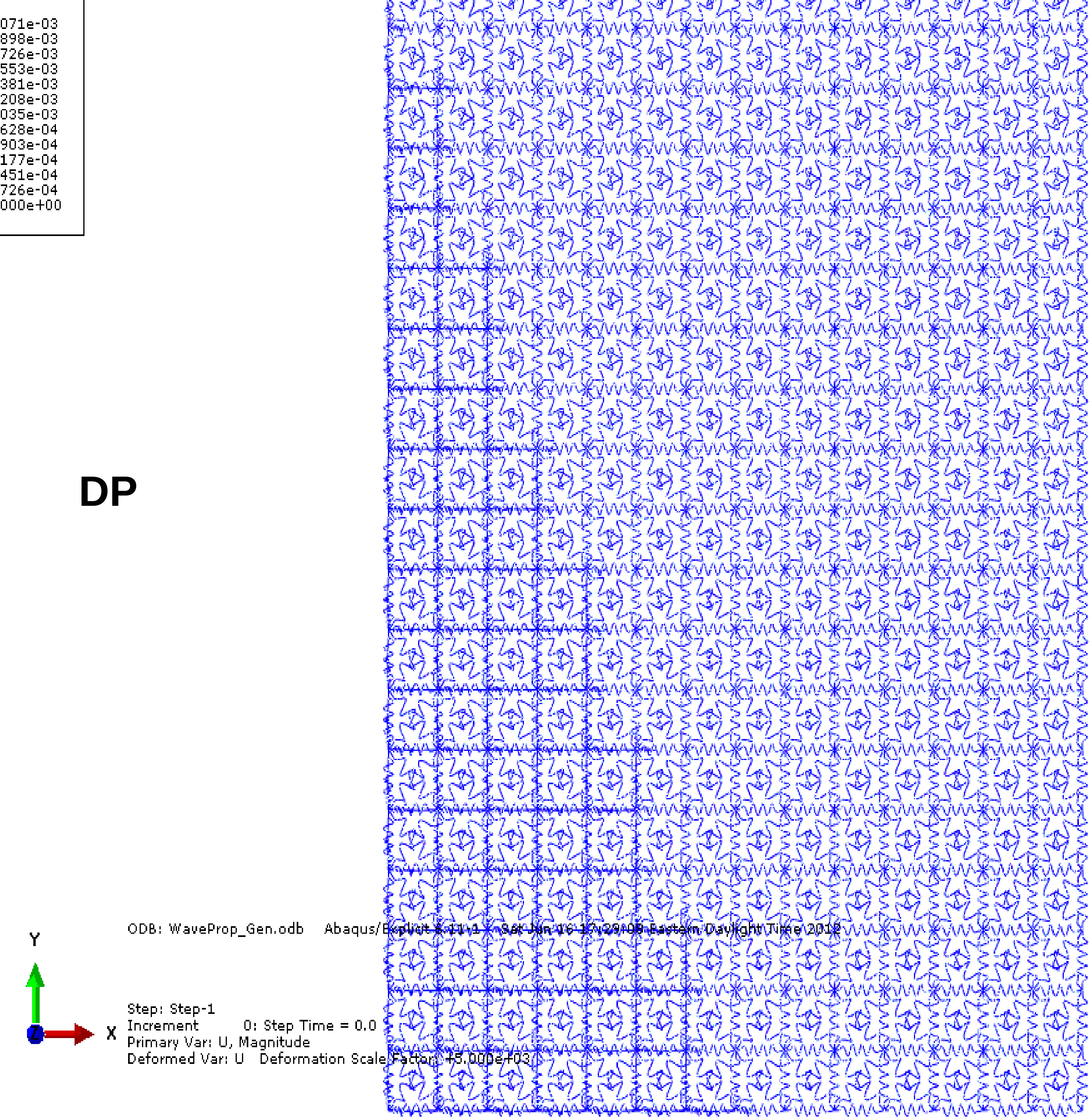
+2.259e-03
+2.054e-03
+1.848e-03
+1.643e-03
+1.437e-03
+1.232e-03
+1.027e-03
+8.214e-04
+6.161e-04
+4.107e-04
+2.054e-04
+0.000e+00
Step: Step-1 Frame:
Total Time: 0.000000
DN
ODB: WaveProp_Gen.odb Abaqus/Explicit 6.11-1 Sat Jun 16 16:29:30 Eastern Daylight Time 2012
Y
X
Step: Step-1
Increment 0: Step Time = 0.0
Primary Var: U, Magnitude
Deformed Var: U Deformation Scale Factor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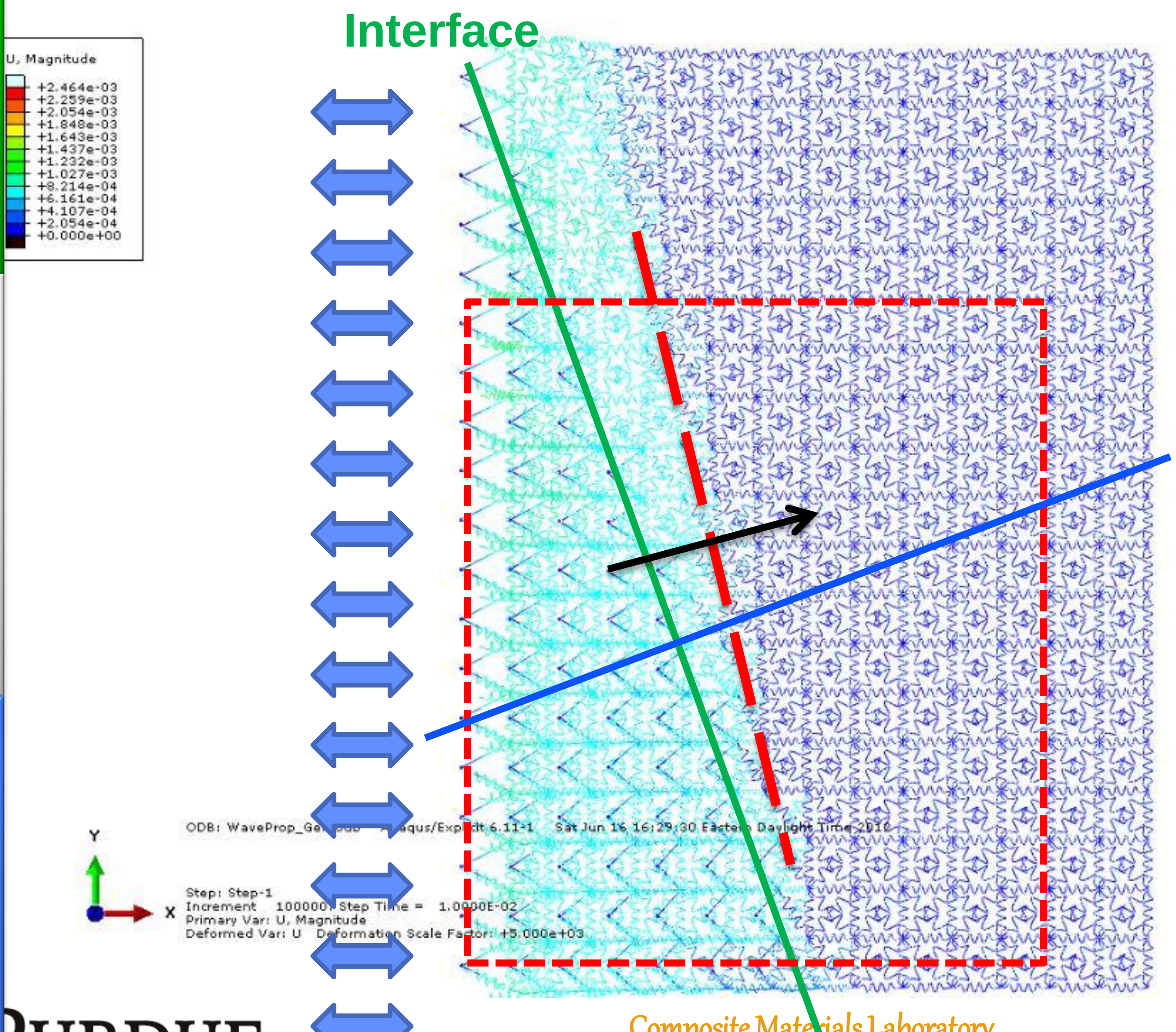
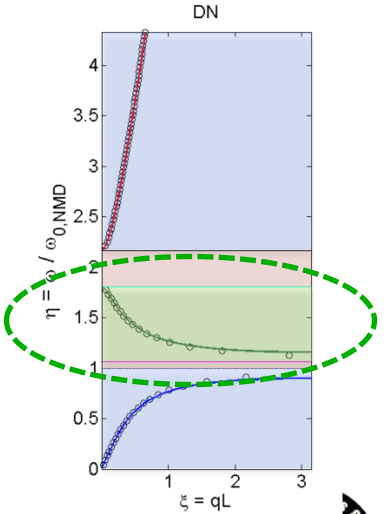
035e-03
628e-04
903e-04
177e-04
451e-04
726e-04
000e+00
DP
ODB: WaveProp_Gen.odb Abaqus/Explicit 6.11-1 Sat Jun 16 17:29:08 Eastern Daylight Time 2012
Y
X
Z
Step: Step-1
Increment 0: Step Time = 0.0
Primary Var: U, Magnitude
Deformed Var: U Deformation Scale Factor: +5.000e+03

Simulation: Plane wave (DN region: 1)

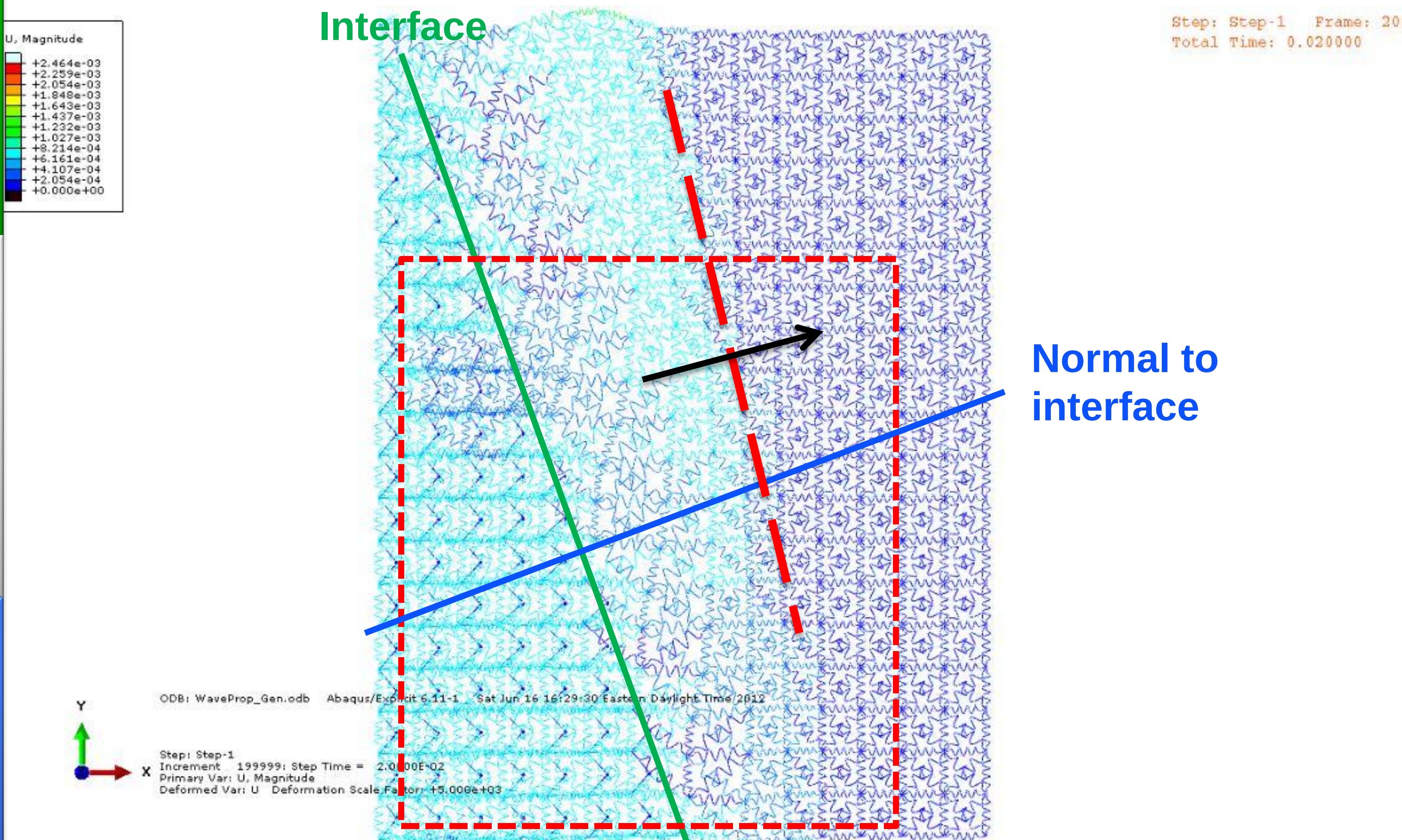




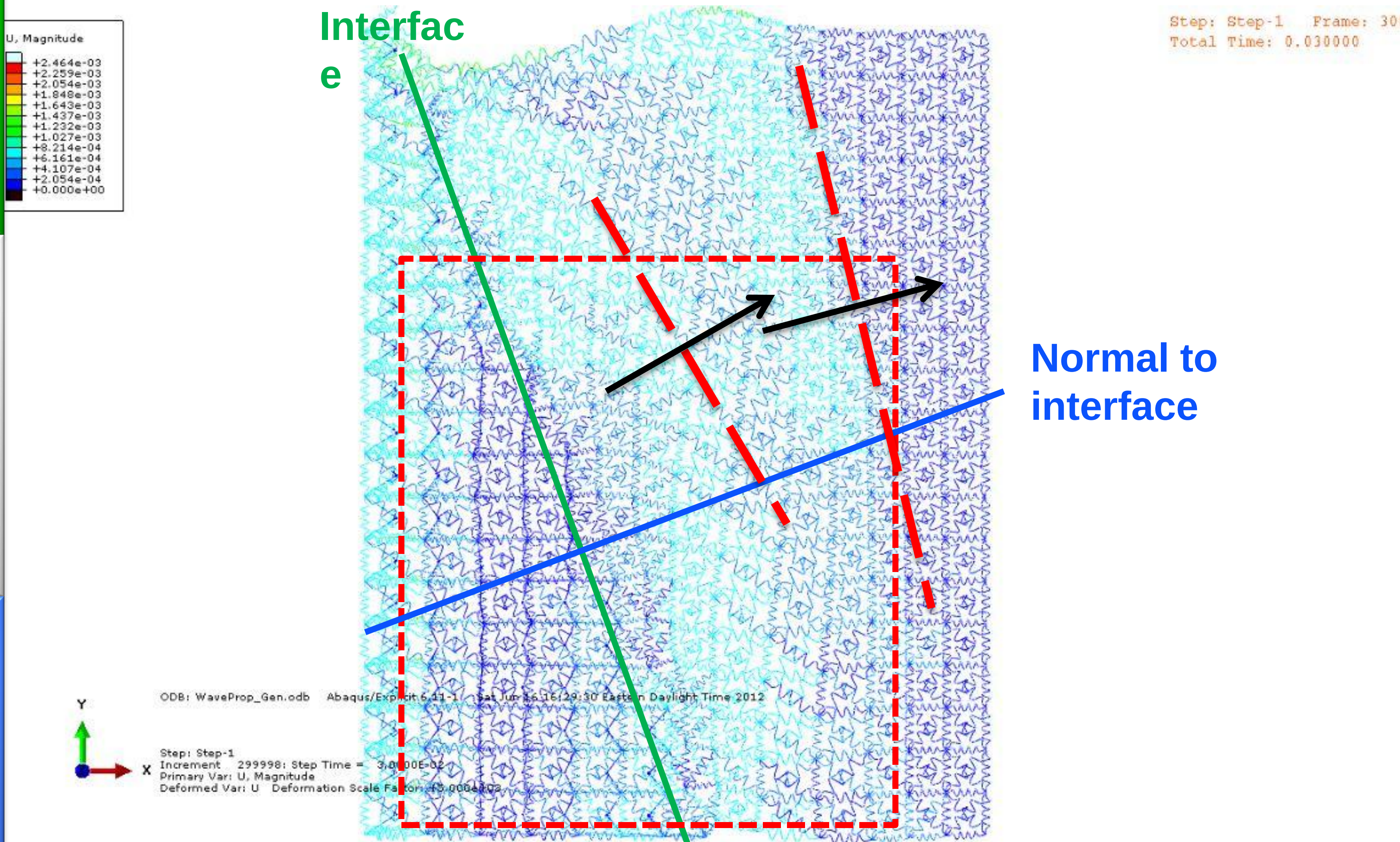
Composite Materials Laboratory

School of Aeronautics and Astronautics

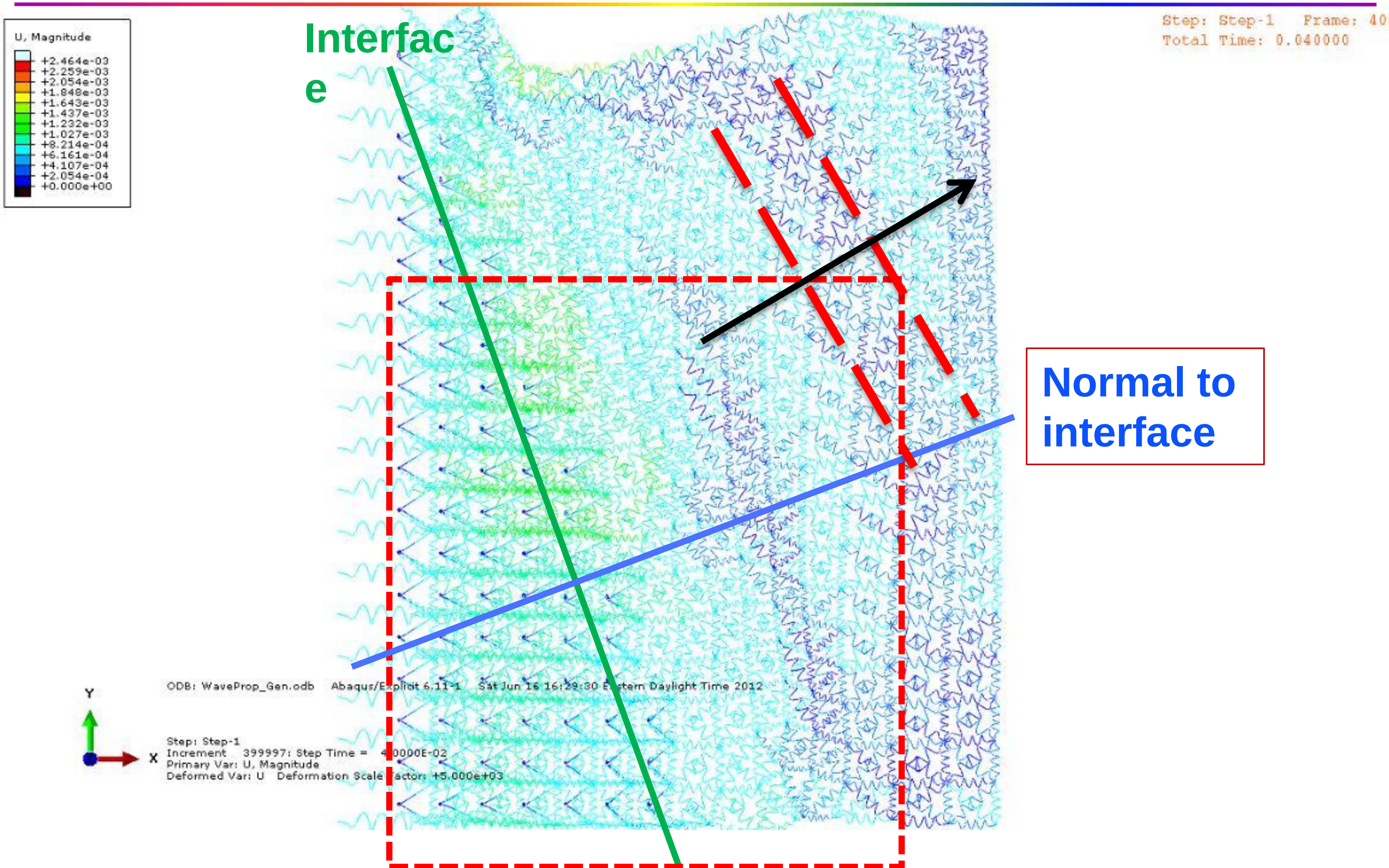
Simulation: Plane wave (DN region: 2)



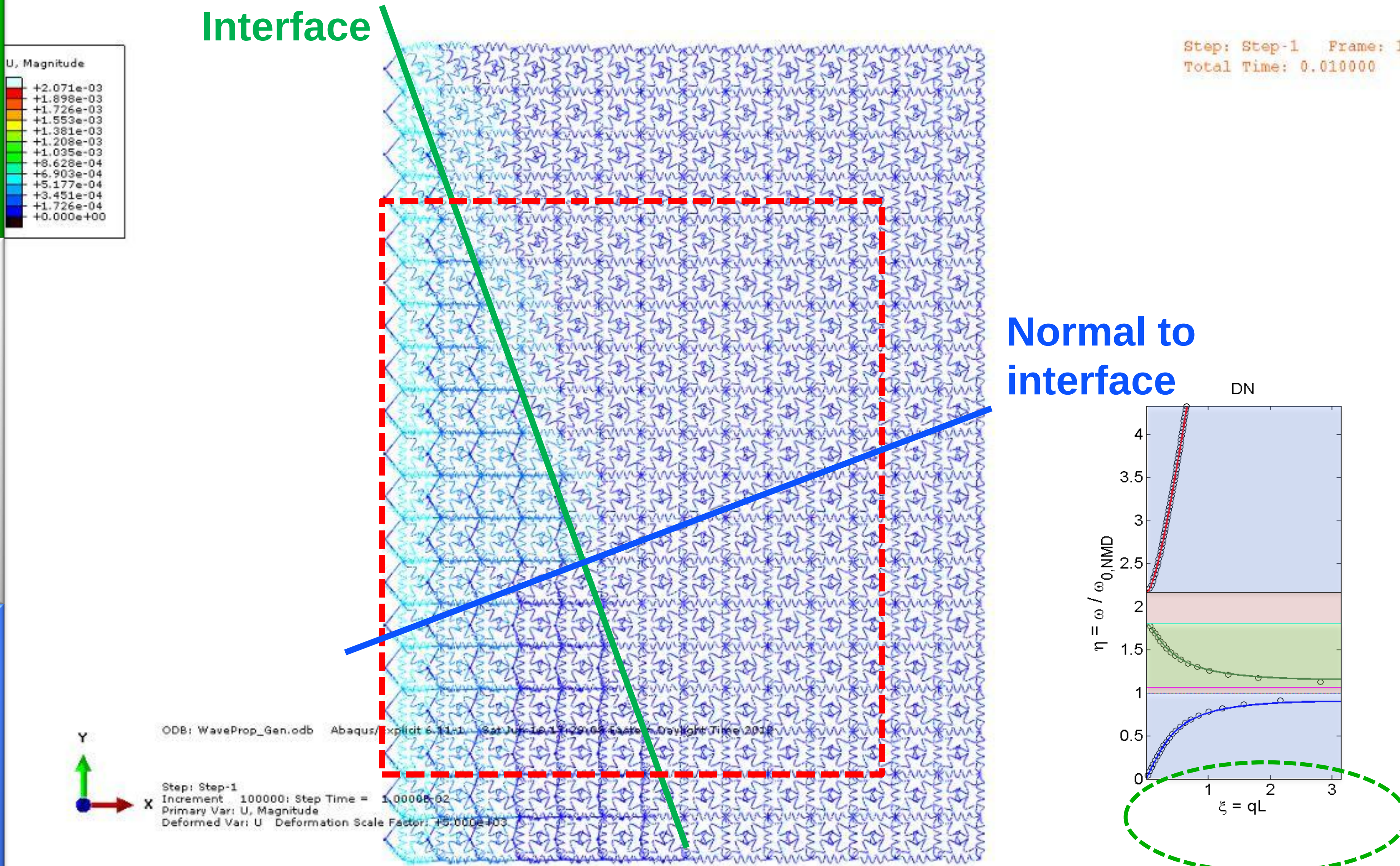
Simulation: Plane wave (DN region: 3)



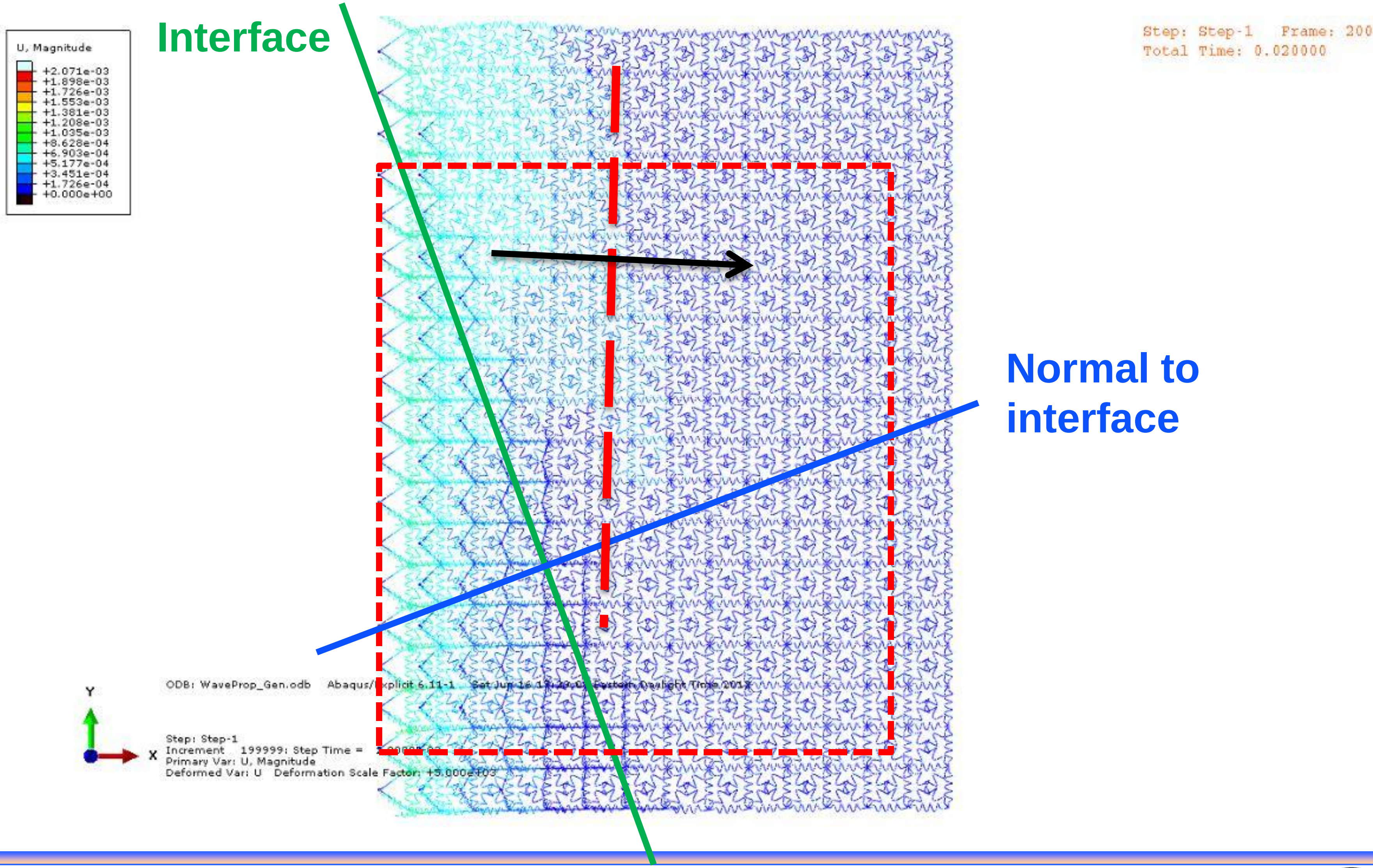
Simulation: Plane wave (DN region: 4)



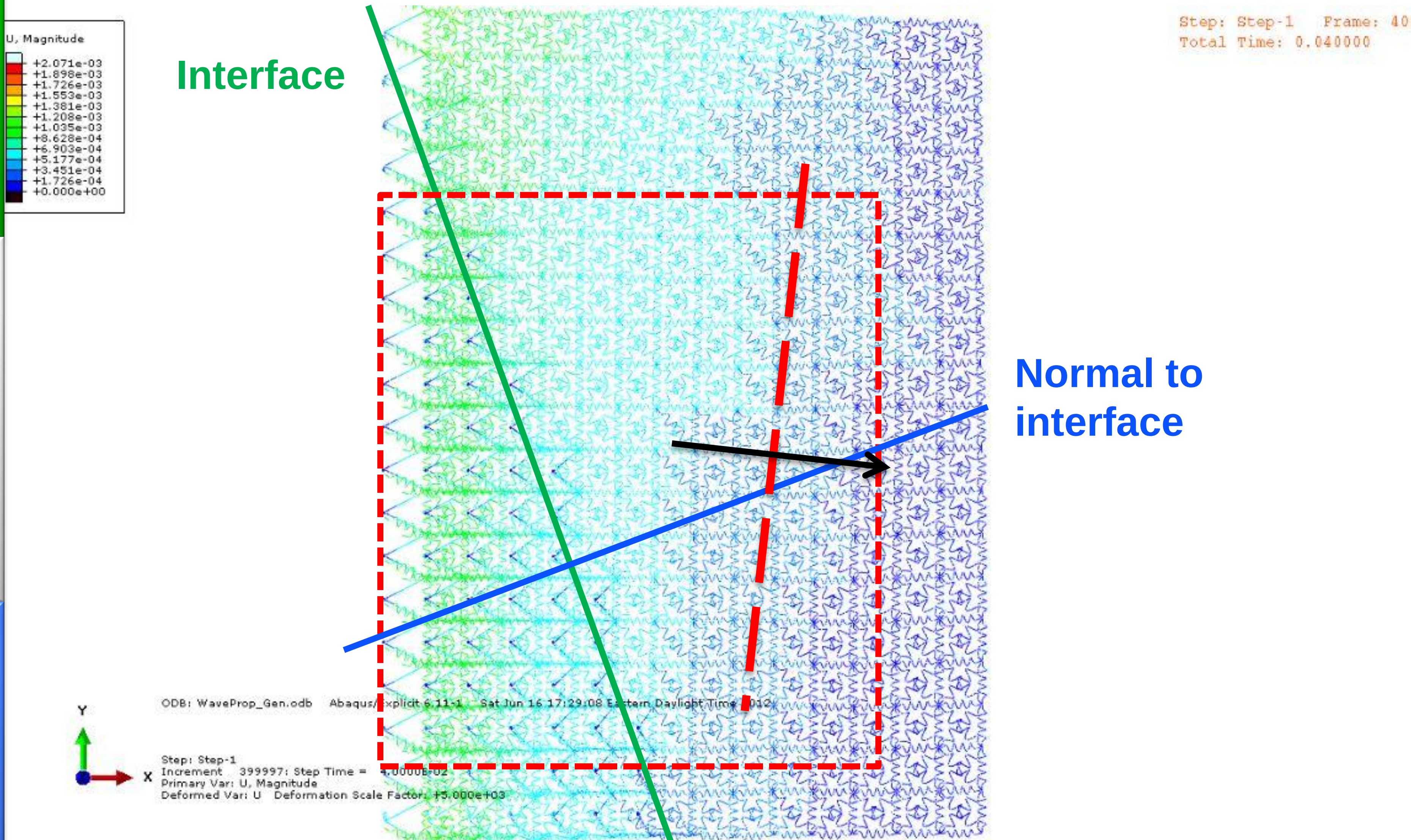
Simulation: Plane wave (DP region: 1)



Simulation: Plane wave (DP region: 2)

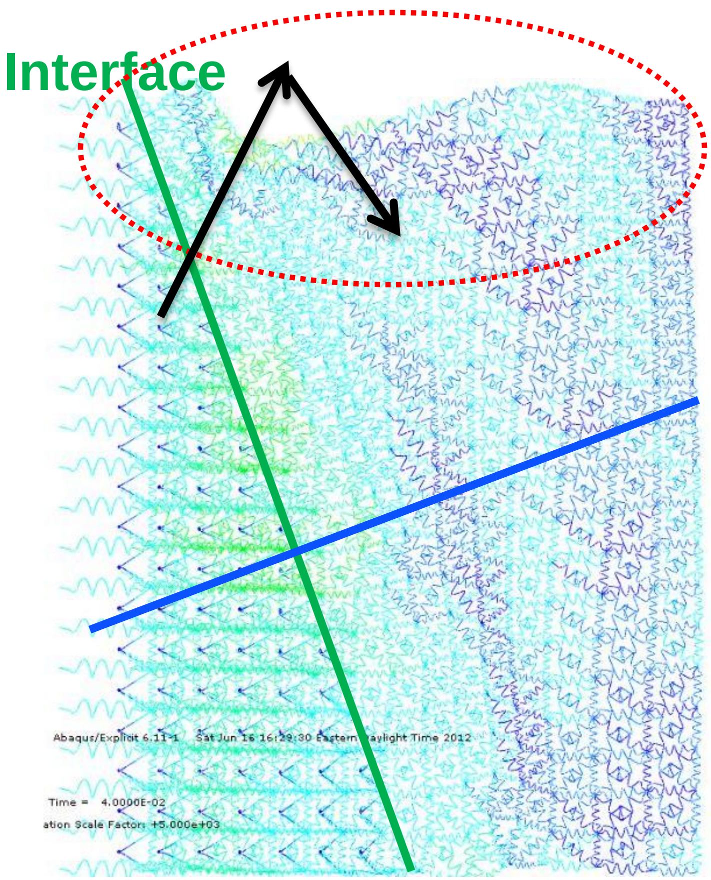


Simulation: Plane wave (DP region: 4)

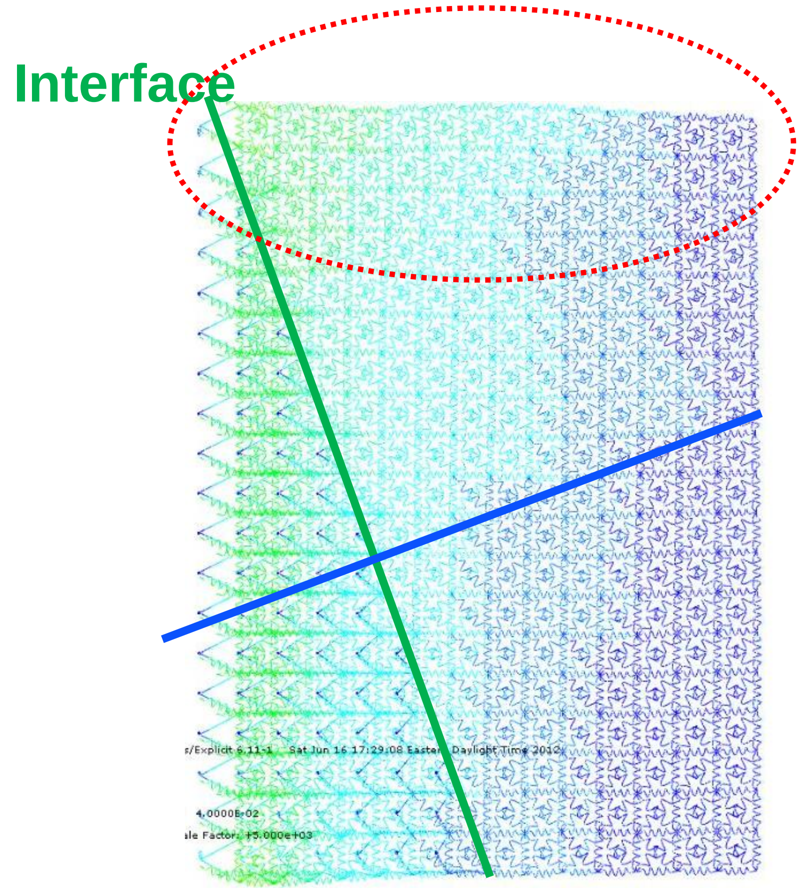


Plane Wave Comparison: DN vs. DP

Double Negativity



Double Positivity



List of Publications

- H. H. Huang and C. T. Sun, “Locally Resonant Acoustic Metamaterials with 2D Anisotropic Effective Mass Density,” *Philosophical Magazine*, Vol. 91, No.6, 2011, pp. 981-996.
- H. H. Huang and C. T. Sun, “A study of Band-gap Phenomena of Two Locally Resonant Acoustic Metamaterials,” *J. Nanoengineering and Nanosystems*, 2011.
- X. N. Liu, G. K. Hu, C.T. Sun, and G. L. Huang, “Wave Propagation Characterization and Design of Two-Dimensional Elastic Chiral Metacomposite,” J*. of Sound and Vibration*, 330, pp. 2536-2553, 2011
- X.N. Liu, G. K. Hu, G. L. H*uang,* and C.T. Sun, “An Elastic Metamaterial with Simultaneously Negative Mass Density and Bulk Modulus,” *Applied Physics Letters,* 98, 251907, 2011.
- H.H. Huang and C.T. Sun, “Behavior of an Acoustic Metamaterial with Extreme Young’s Modulus,” *J. Mechanics and Physics Solids,* doi:10.1016/j.jmps.2011.07.002, 2011.
- R. Zhu, G. L. Huang, H.H. Huang, and C. T. Sun, “Experimental and Numerical Study of Guided Wave Propagation in a Thin Metamaterial Plate,” *Physics Letters A*, 375, , 2011, pp. 2863-2867
- H.H. Huang and C.T. Sun, “Continuum Modeling of a Composite Material with Internal Resonators,” *Mechanics of Materials*, 46, 2012, pp.1-10.
- Hsin-Haou Huang and C. T. Sun, **"**Anomalous Wave Propagation in a One-dimensional Acoustic Metamaterial Having Simultaneously Negative Mass Density and Young's Modulus,” to appear in the Journal of the Acoustical Society of America, 2012